VOS: A New Method for Visualizing Similarities between Objects

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Introduction

- A new method for visualizing similarities between objects is presented.
- The method is called VOS, which is an abbreviation for *visualization of similarities*.
- The objective of VOS is to provide a low-dimensional visualization in which objects are located in such a way that the distance between any pair of objects reflects their similarity as accurately as possible.
- The relationship between VOS and multidimensional scaling (MDS) is established.

Mathematical notation

- n: number of objects
- m: number of dimensions of the visualization
- $\mathbf{S} = (s_{ij})$: $n \times n$ similarity matrix satisfying $s_{ij} \geq 0$, $s_{ii} = 0$, and $s_{ij} = s_{ji}$ for all $i, j \in \{1, \ldots, n\}$
- $\mathbf{X}$: $n \times m$ matrix containing the coordinates of the objects $1, \ldots, n$
- $\mathbf{x}_i$: ith row of $\mathbf{X}$, containing the coordinates of object i

Constrained optimization problem

A VOS solution is obtained by solving the following constrained optimization problem:

Minimize

$$E(\mathbf{X}; \mathbf{S}) = \sum_{i<j} s_{ij} \|\mathbf{x}_i - \mathbf{x}_j\|^2 \tag{1}$$

subject to

$$\sum_{i<j} \|\mathbf{x}_i - \mathbf{x}_j\| = 1, \tag{2}$$

where $\| \cdot \|$ denotes the Euclidean norm.

Ideal coordinates

- The ideal coordinates of object i are given by

$$c_i(\mathbf{X}, \mathbf{S}) = \frac{\sum_j s_{ij} \mathbf{x}_j}{\sum_j s_{ij}}. \tag{3}$$

- Suppose the coordinates of all objects except object i are fixed. The objective function of VOS then reduces to

$$E_i(\mathbf{x}_i; \mathbf{X}, \mathbf{S}) = \sum_j s_{ij} \|\mathbf{x}_i - \mathbf{x}_j\|^2. \tag{4}$$

- Minimization of (4) results in

$$\mathbf{x}_i = c_i(\mathbf{X}, \mathbf{S}). \tag{5}$$

- Apparently, VOS has the tendency to locate objects close to their ideal coordinates.

A simple example data set

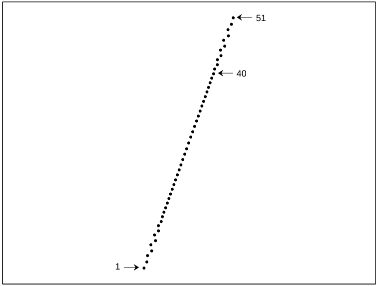
Data set taken from Mardia, Kent, and Bibby (1979)[1]:

- $n = 51$
- $s_{ij} = \begin{cases} 8 & \text{if } 1 \leq |i-j| \leq 3 \\ 7 & \text{if } 4 \leq |i-j| \leq 6 \\ \multicolumn{2}{c}{\dots\dots\dots\dots\dots\dots} \\ 1 & \text{if } 22 \leq |i-j| \leq 24 \\ 0 & \text{otherwise} \end{cases}$

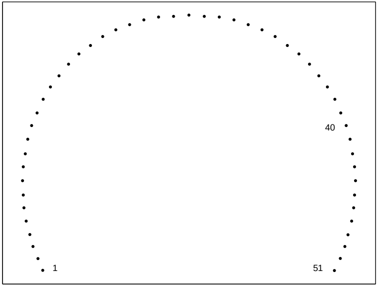
[1] K.V. Mardia, J.T. Kent, and J.M. Bibby. *Multivariate analysis*. Academic Press, 1979.

Results of VOS and MDS

VOS



Ordinal MDS (PROXSCAL)



- The MDS solution demonstrates the horseshoe effect (Mardia et al., 1979).
- VOS seems to take into account indirect similarities via third objects.

Sammon mapping

Objective

- To locate objects in a low-dimensional space in such a way that the distance between any pair of objects reflects their dissimilarity as accurately as possible.

Mathematical notation

- $\mathbf{D} = (d_{ij})$: $n \times n$ dissimilarity matrix satisfying $d_{ij} > 0$ and $d_{ij} = d_{ji}$ for all $i, j \in \{1, \ldots, n\}$

Optimization problem

- A Sammon mapping solution is obtained by minimizing the following objective function

$$\sigma(\mathbf{X}; \mathbf{D}) = \sum_{i<j} \frac{(d_{ij} - \|\mathbf{x}_i - \mathbf{x}_j\|)^2}{d_{ij}}. \tag{6}$$

Relationship between VOS and MDS

Conditional equivalence of VOS and Sammon mapping

Theorem

Let $s_{ij} > 0$ for all i and j ($i \neq j$), and let similarities be transformed into dissimilarities using $d_{ij} = s_{ij}^{-1}$ ($i \neq j$). VOS and Sammon mapping are then equivalent in the sense that VOS solutions and Sammon mapping solutions differ only by a multiplicative constant.

A proof is provided in Van Eck and Waltman (2006)[2].

Conditional equivalence of VOS and weighted MDS

Sammon mapping is equivalent to weighted MDS where for each pair of objects i and j the weight equals d_{ij}^{-1}. It therefore follows from the above theorem that there also exists a conditional equivalence between VOS and weighted MDS.

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[2] N.J. van Eck and L. Waltman. VOS: a new method for visualizing similarities between objects. Technical Report ERS-2006-020-LIS, Erasmus University Rotterdam, Erasmus Research Institute of Management, 2006.

Conclusions

VOS has the following properties:

- VOS has the tendency to locate objects close to their ideal coordinates.
- VOS seems to pay more attention to indirect similarities via third objects than MDS.
- VOS is, under certain conditions, equivalent to Sammon mapping and, as a consequence, to weighted MDS.

References

Description and analysis of VOS

- N.J. van Eck and L. Waltman. VOS: a new method for visualizing similarities between objects. Technical Report ERS-2006-020-LIS, Erasmus University Rotterdam, Erasmus Research Institute of Management, 2006.

Application of VOS to the visualization of associations between concepts based on co-occurrence data

- N.J. van Eck, L. Waltman, J. van den Berg, and U. Kaymak. Visualizing the WCCI 2006 knowledge domain. In *Proceedings of the 15th IEEE International Conference on Fuzzy Systems*, 2006. Accepted for publication.

Related poster presentation

For a practical application of VOS and an experimental comparison between VOS and MDS, please visit the poster presentation *A Comparison of Knowledge Domain Visualization Approaches* by Nees Jan van Eck.